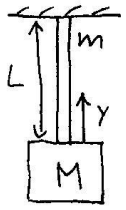
Zadanie 1



(a) $v(y=0) = \pm\sqrt{\frac{Mg}{m/L}}$ $\qquad$ $v(y=L) = \pm\sqrt{\frac{(M+m)g}{m/L}}$

(b) I

$$\begin{cases} v(y) = \pm\sqrt{\frac{(M + \frac{m}{L}y)g}{m/L}} = \pm\sqrt{\frac{Mg}{m/L} + gy} \\ a = \text{constant} \rightarrow v(y) = \pm\sqrt{v^2(t=0) + 2a\Delta y} \end{cases}$$

$$a = \frac{g}{2}$$

(b) II

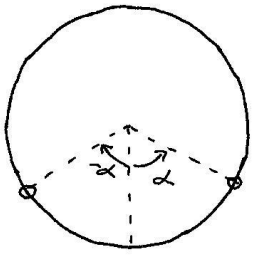
$$a = \frac{dv}{dt} = \frac{dv}{dy}\frac{dy}{dt}$$

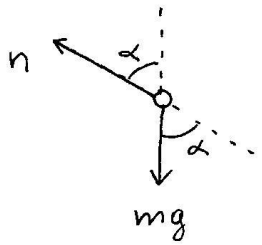
$$= \frac{d}{dy}\left(\pm\sqrt{\frac{Mg}{m/L} + gy}\right) v$$

$$= \pm \frac{g}{2\sqrt{\frac{Mg}{m/L} + gy}} v$$

$$= \frac{g}{2v} v = \frac{g}{2} \qquad \square$$

Zadanie 2





$$\begin{cases} n\cos\alpha = mg \\ n - mg\cos\alpha = m\dfrac{v^2}{R} \\ mg(R + R\cos\alpha) = m\dfrac{v^2}{2} \end{cases}$$

$$3\cos^2\alpha + 2\cos\alpha - 1 = 0$$

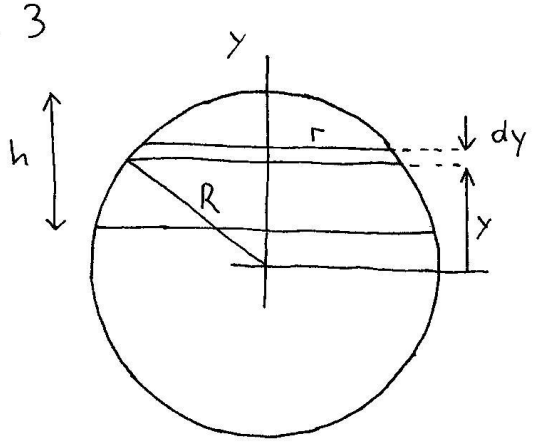
$$\cos\alpha = \frac{1}{3}$$

$$a = \frac{n\sin\alpha}{m} = \frac{mg\sin\alpha}{\cos\alpha\, m}$$

$$= \frac{\sqrt{1-\cos^2\alpha}}{\cos\alpha}\, g = \sqrt{8}\, g \qquad \square$$

Zadanie 3

(a)



$$dV = \pi r^2 dy = \pi (R^2 - y^2) dy$$

$$V_{cz} = \int_{R-h}^{R} \pi (R^2 - y^2) dy = \pi R^2 \int_{R-h}^{R} dy - \pi \int_{R-h}^{R} y^2 dy$$

$$= \pi R^2 \left[R - (R-h) \right] - \pi \left(\frac{1}{3} y^3 \right) \Big|_{R-h}^{R}$$

$$= \pi R^2 h - \frac{1}{3} \pi \left[R^3 - (R-h)^3 \right]$$

$$V_{cz} = \frac{1}{3} \pi h^2 (3R - h)$$

(b)

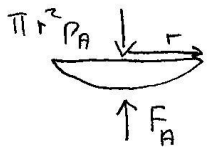
$$(V_K - V_{cz}) \rho_w g = V_K \rho_K g$$

$$\rho_K = \left(1 - \frac{V_{cz}}{V_K} \right) \rho_w$$

$$\rho_K = \left(1 - \frac{\frac{5}{24}}{\frac{4}{3}} \right) \rho_w = \frac{27}{32} \rho_w$$

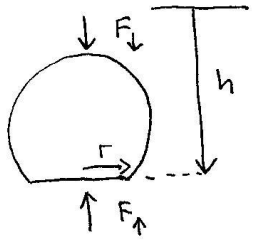
Zadanie 3 c.d.

(c) czasza "zanurzona" w powietrzu



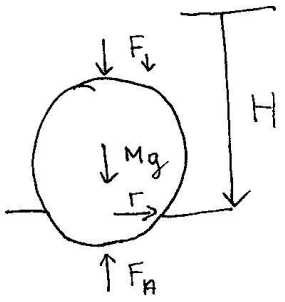
$F_A = \pi r^2 p_A$

kula bez czaszy zanurzona w wodzie



$F_\uparrow - F_\downarrow = (V_K - V_{cz}) \rho_w g$

$F_\uparrow = \pi r^2 (p_A + \rho_w h g)$



nacisk $= F_\downarrow + Mg - F_A$

$= \pi r^2 (p_A + \rho_w H g) - (V_K - V_{cz}) \rho_w g$

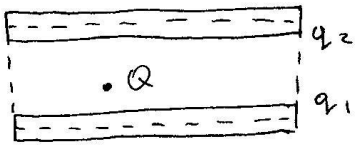
$+ V_K \left(1 - \frac{V_{cz}}{V_K}\right) \rho_w g - \pi r^2 p_A$

$= \pi r^2 \rho_w H g = \pi \left[R^2 - \left(\frac{R}{2}\right)^2\right] \rho_w H g$

$= \frac{3}{4} \pi R^2 \rho_w H g$ □

Zadanie 4

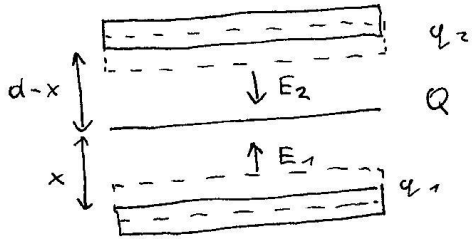
(a)



$$Q + q_1 + q_2 = 0$$

$$q_1 + q_2 = -Q$$

(b)



$$\begin{cases} E_1 A = \dfrac{q_1}{\epsilon_0} \\ E_2 A = \dfrac{q_2}{\epsilon_0} \\ E_1 x = E_2 (d-x) \\ q_1 + q_2 = -Q \end{cases}$$

$$q_1 = -\left(1 - \frac{x}{d}\right) Q$$

$$q_2 = -\frac{x}{d} Q$$

□